To **solve** an equation, we need to get the \( x \) by itself — when it’s by itself, we get the answer of what \( x \) is! In order to get the \( x \) by itself, we need to **rearrange** the numbers and symbols in the equation while still keeping the equation accurate.

Think of the equals sign in the equation as a balance scale. We can change the positions of items on the scales, and take items on or off — we can change the position of numbers and symbols in the equation, and remove numbers or add them on — as long as we keep the scales balanced. We keep the scales balanced by **always doing the same thing to both sides of the equation**.

Remember also that the sign of a variable or constant is what is in **front** of it; sometimes it’s an “invisible +” if it’s at the beginning.

To **solve** an equation, **rearrange** so that all variable parts (anything with \( x \) in) are on one side of the equal sign, and all number parts (parts with just numbers, not \( x \)’s) are on the other side. To do this rearranging, you need to identify what operations are being used (Add, Subtract, Multiply, Divide) and “Undo” operations by using opposite operations. **Remember: Whatever you do to one side, you must do to the other side to keep equation balanced.**

**Example 1:** Solve \( 4x - 5 = 15 \)

\[
\begin{align*}
4x - 5 &= 15 \\
+5 &+5
\end{align*}
\]

- **Add 5 to both sides** because this will remove the minus 5 from the LHS and leave just the \( 4x \) (add and subtract are opposite operations).

\[
\begin{align*}
4x &= 20 \\
\div4 &\div4
\end{align*}
\]

- **Simplify** — notice there’s now only \( x \) parts on LHS and number parts on RHS.

\[
\begin{align*}
\frac{4x}{4} &= \frac{20}{4} \\
\end{align*}
\]

- **Divide both sides by 4** because 4 is multiplied by \( x \), so the opposite operation — division by 4 — will remove the 4 and leave only \( x \).

\[
\begin{align*}
x &= 5
\end{align*}
\]

- **Simplify** — notice we now have the \( x \) by itself and our **answer is** \( x = 5 \).

Let’s check our answer in the original problem by replacing \( x \) with 5: \( 4 \times 5 - 5 = 20 - 5 = 15 \).
The most important rule to remember is to do the same thing to both sides of the equation. This preserves equality.

Example 2: Solve \( \frac{x}{3} + 4 = 9 \)

1. Subtract 4 from both sides because add and subtract are opposite operations, so subtracting 4 removes plus 4 from LHS and leaves just \( \frac{x}{3} \).

\[
\frac{x}{3} + 4 = 9
\]

\[
\frac{x}{3} = 5
\]

2. Simplify – notice there’s now only \( x \) parts on LHS and number parts on RHS.

\[
\frac{x}{3} \times 3 = 5 \times 3
\]

3. Multiply by 3 on both sides because \( x \) is divided by 3, so the opposite operation – multiplication by 3 – will remove the 3 and leave only \( x \).

\[
x = 15
\]

Let’s check our answer in the original problem by replacing \( x \) with 15: \( \frac{15}{3} + 4 = 5 + 4 = 9 \).

Example 3: Solve \( \frac{5 + 3x}{2} + 5 = 3x \)

1. Subtract 5 from both sides because add and subtract are opposite operations. So subtracting 5 removes plus 5 from LHS and leaves just \( \frac{5 + 3x}{2} \).

\[
\frac{5 + 3x}{2} + 5 = 3x
\]

\[
\frac{5 + 3x}{2} = 3x - 5
\]

2. Multiply by 2 on both sides because \( 5 + 3x \) is divided by 2, so the opposite operation – multiplication by 2 – will remove the 2 and leave only \( 5 + 3x \). NOTE: \( \frac{2}{1} \) is the same as 2, since 2 divided by 1 equals 2.

\[
\frac{5 + 3x}{2} \times \frac{2}{1} = (3x - 5) \times \frac{2}{1}
\]

3. Simplify by multiplying LHS and expanding brackets RHS. Then subtract \( 3x \) from both sides since add and subtract are opposite operations, so subtracting \( 3x \) removes \( +3x \) from LHS.

\[
5 + 3x = 6x - 10
\]

\[
-3x
\]

\[
-3x
\]

4. Add 10 to both sides, removing -10 from RHS and rearranging equation with only \( x \) parts on LHS and number parts on RHS.

\[
5 = 3x - 10
\]

\[
+10
\]

\[
+10
\]

\[
15 = 3x
\]

5. Divide both sides by 3 because 3 is multiplied by \( x \), so the opposite operation will remove the 3 and leave only \( x \).

\[
\frac{15}{3} = \frac{3x}{3}
\]

\[
x = 5
\]

Our answer is: \( x = 5 \)

Let’s check our answer in the original problem by replacing \( x \) with 5: LHS: \( \frac{5 + 3 \times 5}{2} + 5 = \frac{20}{2} + 5 = 15 \), RHS: \( 3 \times 5 = 15 \). Note both sides equal so answer is right.
**Example 4: Solve** \(10y - (4y + 8) = -20\)

- **Step 1:** Distribute -1 on the left side.
  \[
  10y - (4y) - (8) = -20
  \]
- **Step 2:** Simplify.
  \[
  6y = -12
  \]
- **Step 3:** Add 8 to both sides to get \(6y\) by itself.
  \[
  6y + 8 = -12 + 8
  \]
- **Step 4:** Divide both sides by 6 to get \(y\) by itself.
  \[
  y = \frac{-12}{6} = -2
  \]

**Answer:** \(y = -2\)

Let’s check our answer in the original problem by replacing \(y\) with -2:

\[
LHS: 10 \times (-2) - (4 \times (-2) + 8) = -20 - (-8 + 8) = -20 - 0 = -20
\]

Making a variable the subject of an equation

Sometimes a question asks you to make a variable the subject of an equation. This means you need to get a variable by itself on one side of the equals sign, so it’s just like solving an equation. For example, if \(Q = 110 - 4P\), and you are asked to make \(P\) the subject of the equation, the way to do this is just to solve the equation – i.e. to get \(P\) by itself on one side of the equals sign.

**Example 5:** Make \(P\) the subject of \(Q = 110 - 4P\)

- **Step 1:** Subtract 110 from both sides to get \(4P\) by itself.
  \[
  Q - 110 = 4P
  \]
- **Step 2:** Divide both side by 4 to get \(P\) by itself.
  \[
  \frac{Q - 110}{4} = P
  \]

**Answer:** \(P = \frac{Q - 110}{4}\)
Solve:

1. \(2x - 5 = 17\)
2. \(3y + 7 = 25\)
3. \(5n - 2 = 38\)

4. Rearrange this formula \(A = 2a^2 + 4ab\) so that \(b\) is the subject of the formula.

5. \(s = ut + \frac{1}{2}at^2\) is a formula used in Physics to calculate distance. Make "\(a\)" the subject of the formula.

Question 3:
\[n = 8\]

Question 4:
\[
\begin{align*}
\text{Subtract }2a^2\text{ from both sides:} & \quad A - 2a^2 = 4ab \\
\text{Swap sides:} & \quad 4ab = A - 2a^2
\end{align*}
\]

Question 5:
\[
\begin{align*}
\text{Divide both sides by } 2: & \quad \frac{v}{t} = \frac{A}{q} \\
\text{Now, divide both sides by } A: & \quad \frac{1}{A} = \frac{t}{q} \\
\text{And we get:} & \quad a = \frac{v}{t} \\
\end{align*}
\]