SIMULTANEOUS EQUATIONS

Solving by elimination

The solution of a pair of simultaneous equations

Simultaneous equations are solved by finding values of the unknowns which satisfy all of the given equations at the same point (simultaneously). The solution of the pair of simultaneous equations.

$$\mathbf{3x + 2y = 36} \quad \text{and} \quad \mathbf{5x + 4y = 64}$$

Solution: $x = 8$ and $y = 6$

How did we arrive at this solution?

Solve the simultaneous equations:

$\mathbf{3x + 2y = 36}$ *equation (1)*
$\mathbf{5x + 4y = 64}$ *equation (2)*

Step 1:

Notice that when we multiply both sides of equation (1) by 2 we obtain an equivalent equation:

$6x + 4y = 72$ *equation (3)*

Step 2:

When equation (2) is subtracted from equation (3) the terms involving y is eliminated:

$6x + 4y = 72$ *equation (3)*
$\underline{-(5x + 4y = 64)}$ *equation (2)*
$x + 0y = 8$

So, $x = 8$ is part of the solution

Step 3:

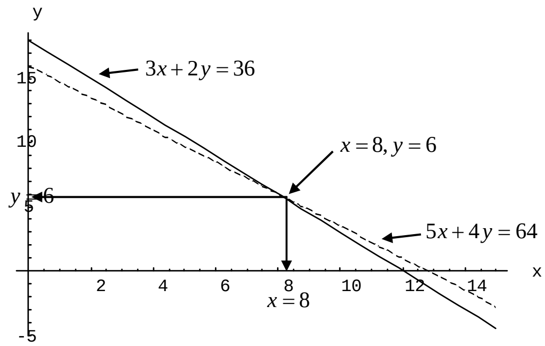
Taking equation (1) *(or if you wish, equation 2)* we substitute the value for x. This will enable us to find the value for y:

$$3(8) + 2y = 36$$
$$24 + 2y = 36$$
$$2y = 36 - 24$$
$$2y = 12$$
$$y = 6$$

Hence, the full solution is: $\mathbf{x = 8, y = 6}$

This is easily verified by substituting these values into the left-hand sides of each equation to obtain the values on the right.
So $x = 8$ and $y = 6$ satisfy the simultaneous equations.

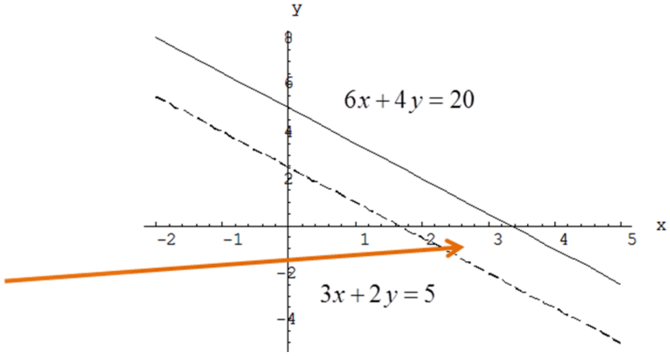
Since any pair of non-parallel lines must cross (i.e. intersect) one another at just one point, each equation must have the same x and y values at that point. Those values provide the solution of the pair of simultaneous equations.



A graphical interpretation of *no* solution!

A pair of simultaneous equations may be solved *only* when the equations are *independent*. For example, it is not possible to solve the simultaneous equations uniquely (that is, just one and only one pair of x and y values that satisfy both equations).

This is because these equations represent parallel lines, and parallel lines do not intersect; they have no common point anywhere, as the diagram shows:



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Let's try another by elimination

You will notice that the idea behind this method is to multiply one (or both) equations by a suitable number so that either the number of y's or the number of x's are the same, so that subtraction eliminates that unknown. It may also be possible to eliminate an unknown by addition, as shown in the next example.

Solve the simultaneous equations:	$\mathbf{5x - 3y = 26}$ *equation (1)* $\mathbf{4x + 2y = 34}$ *equation (2)*	
There are many ways that the elimination can be carried out. In this example y is eliminated.		
Step 1: The number of y's in both equations can be made the same by multiplying equation (1) by 2 and equation (2) by 3. This gives: $10x + 6y = 52$ *(equation 3)* $12x + 6y = 102$ *(equation 4)*	**Step 2:** When these equations are added, we find: $10x - 6y = 52$ *equation (3)* $+\underline{(12x + 6y = 102)}$ *equation (4)* $22x + 0y = 154$ So that: $x = \frac{154}{22} = 7$.	**Step 3:** Substituting the value for x in equation (1) gives: $5(7) - 3y = 26$ $35 - 3y = 26$ $-3y = 26 - 35$ $-3y = -9$ $y = 3$
Hence, the full solution is: $\mathbf{x = 7,\ y = 3}$		

Try these problems:				
	1.	$7x + y = 25$ $5x - y = 11$	5.	$5y + 4z = 32$ $7y - 3z = 62$
	2.	$8x + 9y = 3$ $x + y = 0$	6.	$5a - 2b = 26$ $a + 6b = 18$
	3.	$2x + 13y = 36$ $13x + 2y = 69$	7.	$5x - 4y = 13$ $7x - 6y = 17$
	4.	$7x - y = 15$ $3x - 2y = 19$	8.	$4r - 5t = -23$ $5r - 2t = -16$

1.	$x = 3, y = 4$	2.	$x = -3, y = 3$
3.	$x = 5, y = 2$	4.	$x = 1, y = -8$
5.	$y = 8, z = -2$	6.	$a = 6, b = 2$
7.	$x = 5, y = 3$	8.	$r = -2, t = 3$

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TEL: 61-8-8201 2518
E-MAIL: slc@flinders.edu.au

INTERNET: http://www.flinders.edu.au/SLC
POSTAL: PO BOX 2100, ADELAIDE, SA 5001

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