Student Learning Centre

Logarithms:
in Base 10 (*log*) and the Natural Logarithm (*ln*)

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What is a logarithm?

The logarithm of a number is the power to which another fixed value (the base) must be raised to produce that number. In this worksheet, the focus is on base 10 logarithms, as these are by far the most commonly used, so the base value will always be 10. (See p3 for the Natural Logarithm.)

Example A

$$100 = 10^2$$

- 100 is the number we're finding a logarithm for
- 10 is the base
- 2 is the power (also called index or exponent)

The statement $100 = 10^2$ tells us that the logarithm of 100 is 2, because 2 is the power that we have to raise 10 to in order to get 100.

Often, the statement $100 = 10^2$ is written using logs, as follows: $\log_{10}(100) = 2$

Sometimes, the base 10 is not explicitly written, but just assumed: $\log(100) = 2$

This is read as "the log of 100 is 2".

Example B

$$907 \approx 10^{2.9576}$$

- 907 is the number we're finding a logarithm for
- 10 is the base
- 2.9576 is the power (also called index or exponent)

The statement $907 \approx 10^{2.9576}$ tells us that the logarithm of 907 is approximately 2.9576, because 2.9576 is the power that we have to raise 10 to in order to get 907. (Try putting $10^{2.9576}$ into your calculator and see what you get. Remember it's approximately equal.)

Often, the statement $907 \approx 10^{2.9576}$ is written using logs, as follows: $\log_{10}(907) \approx 2.9756$

Sometimes, the base 10 is not explicitly written, but just assumed: $\log(907) \approx 2.9756$

This is read as "the log of 907 is 2.9756".

More generally, if $a = 10^b$, then $\log(a) = b$.

Why use logarithms?

Logs allows the translation of something that is changing *exponentially* into something that is changing *linearly*. Note in the diagram below how the number we are taking the log of (1, 10, 100, 1000, etc.) is increasing exponentially – the gap between numbers gets bigger each time. In contrast, the logs of those numbers (0, 1, 2, 3, etc.) are increasing linearly:

log *of* 1: $\log_{10}(1) = \log_{10}(10^0) = 0$

log *of* 10: $\log_{10}(10) = \log_{10}(10^1) = 1$

log *of* 100: $\log_{10}(100) = \log_{10}(10^2) = 2$

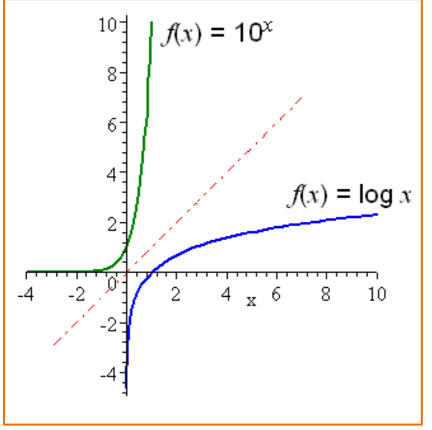
log *of* 1000: $\log_{10}(1000) = \log_{10}(10^3) = 3$

log *of* $10{,}000$: $\log_{10}(10{,}000) = \log_{10}(10^4) = 4$

log *of* $100{,}000$: $\log_{10}(100{,}000) = \log_{10}(10^5) = 5$

log *of* $1{,}000{,}000$: $\log_{10}(1{,}000{,}000) = \log_{10}(10^6) = 6$

Graph of 10^x and $\log(x)$:

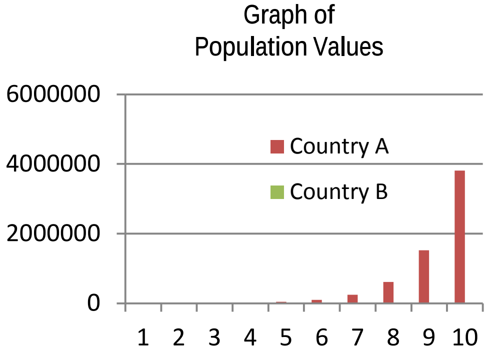


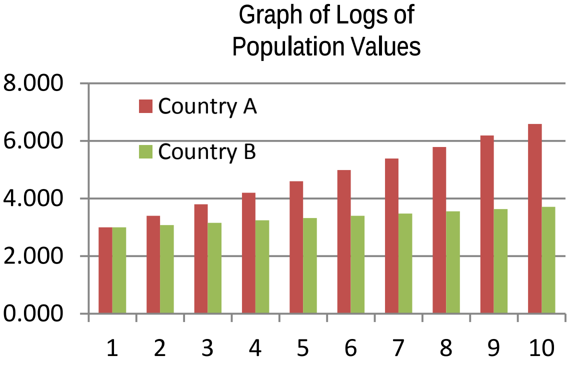
Why use logarithms? *(continued)*

Logs make it easier to compare and analyse quantities that are changing exponentially. For example, consider the graphs below comparing the population growth of two countries. Population is growing in both countries, but much more slowly in Country B than in Country A. The first graph is of the population numbers, and the second graph of the logs of population numbers. Note how the comparison is much easier to make on the second graph of straight lines:

Generation	Country A Population	Country B Population
1	1000	1000
2	2500	1200
3	6250	1440
4	15625	1728
5	39063	2074
6	97656	2488
7	244141	2986
8	610352	3583
9	1525879	4300
10	3814697	5160

Generation	Country A Log of Population	Country B Log of Population
1	3.000	3.000
2	3.398	3.079
3	3.796	3.158
4	4.194	3.238
5	4.592	3.317
6	4.990	3.396
7	5.388	3.475
8	5.786	3.554
9	6.184	3.633
10	6.581	3.713





Laws of Logarithms

The laws of logarithms can simplify what needs to be done when solving problems that involve logarithms.

Law	Definition	Example
Product	$\log a + \log b = \log ab$	$\log 9 + \log 2 = \log 18$
Quotient	$\log a - \log b = \log \frac{a}{b}$	$\log 15 - \log 5 = \log \frac{15}{5} = \log 3$
Power	$\log a^m = m \log a$	$\log 8^3 = 3 \log 8$
Identity	$10^{(\log m)} = m$	$10^{(\log 42)} = 42$
Equality	If $\log a = \log b$ Then $a = b$	If $\log(3x-2) = \log(4x+1)$ Then $3x - 2 = 4x + 1$

What is the Natural Logarithm (ln)?

The letter *e* stands for the number 2.718 … and is a real number constant that appears in some kinds of mathematics problems. Examples of such problems are those involving growth or decay (including compound interest), the statistical "bell curve", and the shape of a hanging cable. It also shows up in calculus quite often, wherever you are dealing with either logarithmic or exponential functions.

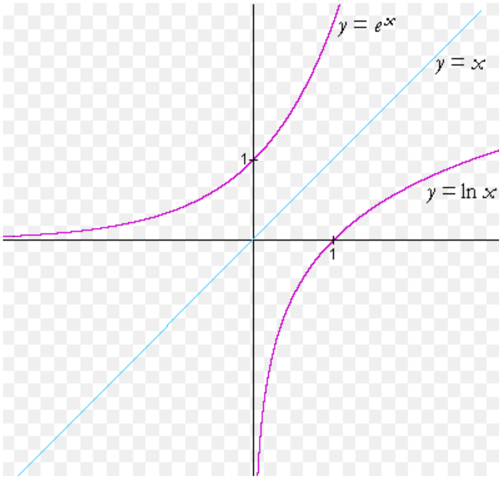
Log_e (log to the base of *e*) is used frequently and is usually written as *ln*, which stands for "natural logarithm".

Note that $\log_e(e) = \log_e(e^1) = 1 = \ln(e)$.

When using *ln*, instead of 10 being the base for logarithmic calculations, *e* is the base. So we have:

		Note approximate values of *e* raised to given power:	
$\ln\ of\ e^0:$	$\log_e(e^0) = \ln(e^0) = 1$	$e^0 =$	1
$\ln\ of\ e^1:$	$\log_e(e^1) = \ln(e^1) = 1$	$e^1 \approx$	2.7183
$\ln\ of\ e^2:$	$\log_e(e^2) = \ln(e^2) = 2$	$e^2 \approx$	7.1891
$\ln\ of\ e^3:$	$\log_e(e^3) = \ln(e^3) = 3$	$e^3 \approx$	20.0855
$\ln\ of\ e^4:$	$\log_e(e^4) = \ln(e^4) = 4$	$e^4 \approx$	54.5981
$\ln\ of\ e^5:$	$\log_e(e^5) = \ln(e^5) = 5$	$e^5 \approx$	148.4132

Graph of $y = e^x$ and $y = \ln(x)$:



In practice, whenever you see *ln(x)* or (without brackets) *ln x*, it should be clear that the natural logarithm is implied. And whenever you see *log(x)* or *log x* – that is, without the base explicitly shown as a subscript – you should read this as the base 10 logarithm, $log_{10}(x)$.

Logs and Your Calculator

Your calculator will be able to calculate logarithms to bases 10 and e (and possibly more).

Usually, the *log* button is used for base 10, and the *ln* button used for base e.

Examples

Using your calculator, find:

$\log_{10}(73)$	(2) ln(5.64)	(3) $e^{1.7299}$
log(0.47)	(5) $\log_e(0.16)$	

Two Other Important Properties

A. log 1 = 0 and ln(1) = 0. That is, the logarithm of 1 to any base is always 0.

B. $\log_{10}10 = 1$ and $\log_e e = \ln(e) = 1$. That is, the logarithm of a number that is the same as the base is always 1.

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Try these out …

Rewrite using the various laws and results discussed in previous sections. The first calculation is done for you. Use your calculator only for the final step, to simplify.

(1) log(3) + log(5)
(2) log(3n) + log(2n)
(3) log108 – log102
(4) log x^2 – log x
(5) log (24)
(6) $\log(2x)^3$
(7) log 1
(8) log(7)
(9) 3log4 – 2log2
(10) $\log(p^3) - \log p^2$
(11) log 32 – log 8
(12) ln(36)

Start with	log(3) + log(5)
Apply log law (product)	log(3) + log(5) = log(3×5)
Simplify with calculator	Log(15) = 1.176 (3 decimal places)

Answers:
1) log(15) = 1.176 (3dp)
2) $\log(6n^2)$
3) log(18/17)=0.025 (3dp)
4) log(x)
5) log(24) = 1.38 (2dp)
6) 3log(2x)
7) log(15) = 1.176 (3dp)
8) log(7) = 0.85 (2dp)
9) 4log2 = 1.20 (2dp)
10) log(p)
11) log(4) = 0.602 (3dp)
12) ln(36) = 3.58 (2dp)

Try these out …

Solve using the various laws and results discussed in previous sections. The first calculation is done for you. There's no need to use a calculator.

(1) log(x(x-2)) = log(x+10)

(2) log(2x+5) = log(x(x+6))

(3) log(7x+9) = log(2x+24)

Start with	Log[(x)(x-2)] = log(x+10)
Apply log law (equality)	$x^2 - 2x = x+10$
Factorise quadratic equation in order to solve it	$x^2 - 2x - x - 10 = 0$ $x^2 - 3x - 10 = 0$ $(x-5)(x+2) = 0$
Solve quadratic equation	x = 5 and x = -2 are solutions (when x = 5, (x-5)=0 and when x = -2, (x+2)=0)

See: http://chilimath.com/algebra/advanced/logs/logeq.html#ex2

Answers:
(2) x=-5 and x=1
(3) x=3

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TEL: 61-8-8201 2518
E-MAIL: slc@flinders.edu.au

INTERNET: http://www.flinders.edu.au/SLC
POSTAL: PO BOX 2100, ADELAIDE, SA 5001