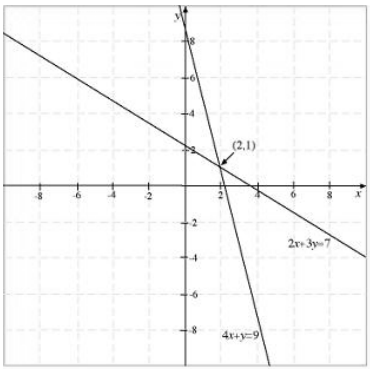
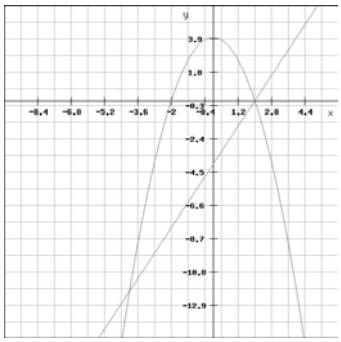
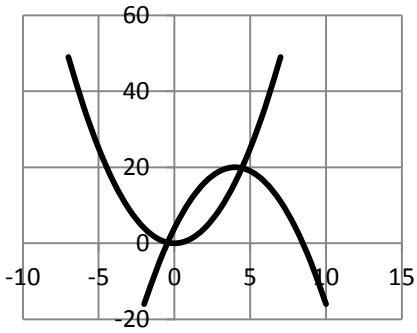
Introduction to Simultaneous Equations

Solving simultaneous equations means finding where two lines meet. The two lines could be two straight lines, or a straight line and a curve, or two curves.





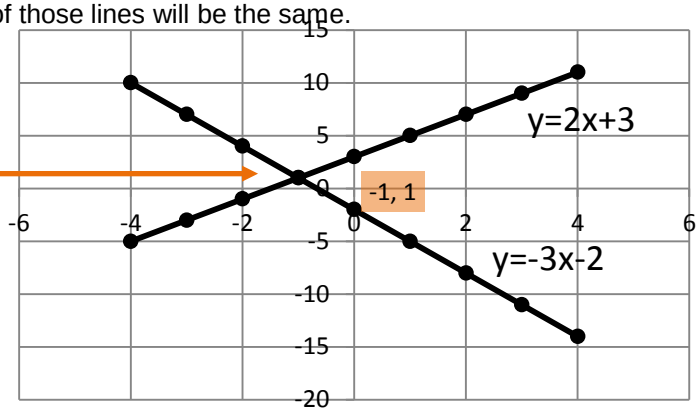


How to Solve Simultaneous Equations

At the point where any two lines meet, the x and y values of those lines will be the same.

Point of intersection is (-1,1) and is a point on both lines.

The solution to this simultaneous equation is (-1,1) or x = -1 and y = 1.



Algebraically, we solve simultaneous equations by setting x or y values equal to each other.
For example: solve $y = 2x + 3$ and $y = -3x - 2$

Step 1: Find x-value at intersection point by setting y-values equal to each other.

$$2x + 3 = 3x - 2$$
$$2x + 3x = -2 - 3$$
$$5x = -5$$
$$\frac{5x}{5} = \frac{-5}{5}$$
$$x = -1$$

Step 2: Substitute x-value found in Step 1 into one equation.

$$y = 2x + 3$$
$$= 2(-1) + 3$$
$$= -2 + 3$$
$$y = 1$$

Step 3 *(optional)*: Double check that values are correct by substituting x-value from Step 1 into the other equation.

$$y = -3x - 2$$
$$= -3(1) - 2$$
$$= 3 - 2$$
$$y = 1$$

Solution: (-1, 1)

Examples To Try (check your answer over the page)

Problem 1: Find where $y = -2x + 7$ and $y = 4x - 5$ cross each other.

Problem 2: Solve $2x + y = 4$ and $-x - 3y = 3$.

Problem 3: Do these lines meet? $y = 3x + 5$ and $y = 3x - 2$.

Problem 4: Find where $y = 4x + 3$ and $y = x^2 + x - 1$ meet.

PROBLEM 1: Find where $y = -2x + 7$ and $y = 4x - 5$ cross each other

Step 1: Find x-value at intersection point by setting equations equal to each other.

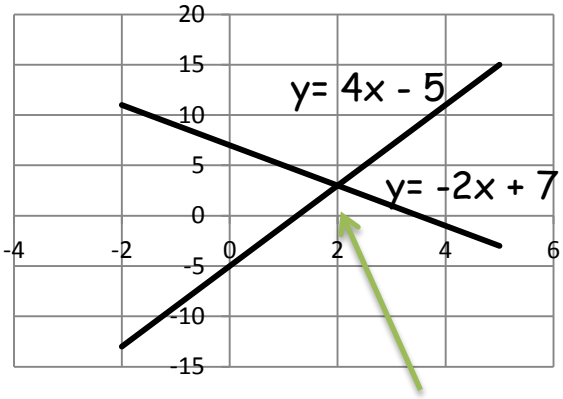
$$-2x + 7 = 4x - 5$$
$$-2x - 4x = -5 - 7$$
$$-6x = -12$$
$$x = 2$$

Step 2: Find y-value at intersection point by substituting x-value at intersection point into one equation.

$$y = -2x + 7$$
$$= -2(2) + 7$$
$$= -4 + 7$$
$$y = 3$$

Step 3 (optional)**:** Double-check y-value is correct by substituting in other equation.

$$y = 4x - 5$$
$$= 4(2) - 5$$
$$= 8 - 5$$
$$y = 3$$



(2,3)

Answer: *Point of intersection is (2, 3)*

PROBLEM 2: Solve $2x + y = 4$ and $-x - 3y = 3$)

Step 1: Rearrange equations in form y = ...

$$2x + y = 4$$
$$y = -2x + 4$$

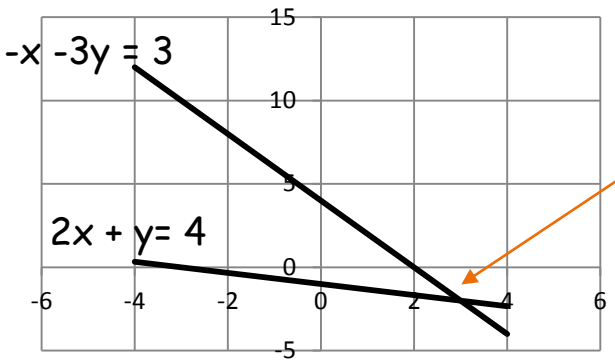
$$-x - 3y = 3$$
$$-3y = 3 + x$$
$$y = \frac{3+x}{-3}$$

Step 2: Find x-value at intersection point by setting equations equal to each other.

$$-2x + 4 = \frac{3+x}{3}$$
$$-3(-2x + 4) = 3 + x$$
$$6x - 12 = 3 + x$$
$$6x - x = 3 + 12$$
$$5x = 15$$
$$x = 3$$

Step 3: Find y-value at intersection point by substituting x-value at intersection point into one equation.

$$y = -2x + 4$$
$$= -2(3) + 4$$
$$= 2$$

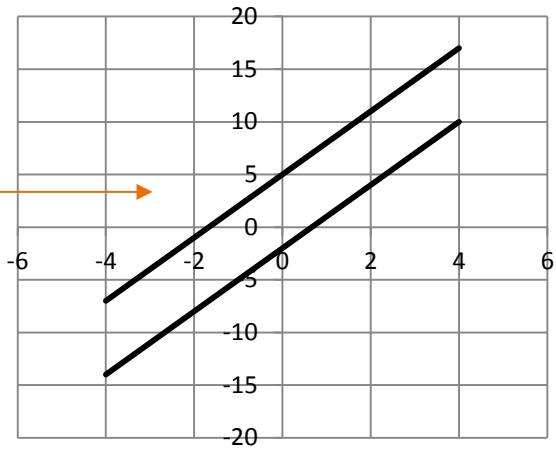


(3,2)

Answer: *Point of intersection is (3, 2)*

PROBLEM 3: Do these lines meet? $y = 3x + 5$ and $y = 3x - 2$

Notice these two lines have the same slope (+3).
This means they are parallel and will never meet.



To confirm this, find x-value at intersection point by setting equations equal to each other:

$$3x + 5 = 3x - 2$$
$$3x - 3x = -2 - 5$$
$$0 = -7$$

Is it ever true that zero equals minus seven?

Answer: These two lines are parallel and never meet.

PROBLEM 4: Find where $y = 4x + 3$ and $y = x^2 + x - 1$ meet.

Step 1: Find x-value at intersection point by setting equations equal to each other.

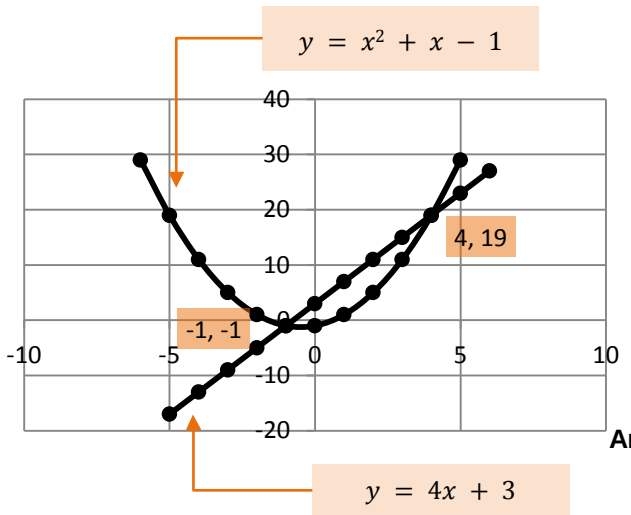
$$4x + 3 = x^2\,x - 1$$
$$0 = x^2 + x - 1 - 4x - 3$$
$$0 = x^2 - 3x - 4$$
$$0 = (x - 4)(x + 1)$$
$$x = 4 \text{ and } x = -1$$

Step 2: Find y-values at intersection point by substituting x-values at intersection point into one equation.

$$y = 4x + 3 \qquad y = 4x + 3$$
$$= 4(4) + 3 \qquad = 4(-1) + 3$$
$$y = 19 \qquad y = -1$$

Step 3: (optional)**:** Double check y-values correct by substituting x-values into other equation.

$$y = (4)2 + 4 - 1 \qquad y = (-1)^2 + (-1) - 1$$
$$= 16 + 4 - 1 \qquad = 1 - 1 - 1$$
$$= 19 \qquad = -1$$



Answer: the line and curve intersect at (-1, -1) and (4, 19).

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